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Power distribution in Czecho–Slovak parliaments[1]

by

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One of the most interesting applications of the theory of games in political problems is an analysis of electoral and voting procedures and power structures in political representative bodies. Electoral and voting procedures, as special cases of group decision problems, are studied by microeconomic theory, mathematical politology and the theory of public choice.

In the present paper we examine a special voting configuration, called multi–cameral legislature, and use some concepts of cooperative game theory (voting games, Shapley value, Shapley–Shubik power index) to estimate an a priori distribution of power among different political formations in federal parliament and national parliaments of the Czecho-Slovak Federal Republic.

[1] This paper was written before the split of Czech and Slovak Republic. It illustrates quit well, though, the difficulties in such multicameral legislatures (eds.)

1. Voting games

Let $I = 1, 2, \ldots, n$ be a set of "players" and $S \subseteq I$ be a coalition of players. A real–valued function $v(S)$ defined for all subsets $S \subseteq I$ such that

$$v(0) = 0$$

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \quad \text{for} \quad S_1, S_2 \subseteq I, S_1 \cap S_2 = \emptyset$$

where $\emptyset$ is the empty set, is termed characteristic function, and a pair (I, v) is called n–person cooperative game in the characteristic function form.

A game (I, v) is said to be simple if for all $S \subseteq I$ either $v(S) = 0$, or $v(S) = 1$.

Let $I = 1, 2, \ldots, n$ be a set of political formations ("parties") in some representative body ("parliament"), a_i be number of deputies of the i–th party,

$$a_0 = \sum_{i \in I} a_i$$

be the total number of deputies and α be a voting rule in the following sense: minimal number of votes necessary for winning in a voting situation (approving a proposal) is

$$\text{int}(\alpha a_0) + 1$$

The game theoretical aspect of the situation is obvious: parties create voting coalitions to collect necessary number of votes to win.

We shall adopt the following simplifying assumptions:

a) All deputies of the same party always vote together.

b) If in a voting situation some parties create a coalition they vote together.

c) Any coalition of parties is possible and all coalitions are equally probable.

We shall say that $S \subseteq I$ is a winning coalition (with respect to a voting rule α), if

$$\sum_{i \in S} a_i - \text{int}(\alpha a_0) > 0 \tag{1}$$

and a losing coalition in opposite case. It is easy to show that the function:

$$v(S) = \begin{cases} 1 & \text{if } S \text{ is a winning coalition} \\ 0 & \text{if } S \text{ is a losing coalition} \end{cases} \tag{2}$$

is a characteristic function. A simple game (I, v, α) with characteristic function v defined as in (1)–(2) is called a voting game.

Let (I, v, α) be a voting game, $S \subseteq I$ and $i \in S$, and let $v(S) = 1,\ v(S-i) = 0$, then we shall say that the player i is essential for coalition S to be winning.

2. The Shapley–Shubik power index

Distribution of votes among the parties in a voting game is not a sufficient characteristic of power or influence distribution. That can be clearly seen from a simple example of the following 3–parties parliament with 100 deputies:

parties	deputies
1	49
2	2
3	49

Take $\alpha = 0.5$. With respect to 50% majority rule all three parties have the same position in the voting game (any two–parties) coalition is a winning one, no single party can win). In fact, under certain circumstances (if two large parties 1 and 3 are on the opposite sides of political spectrum) the role of party 2 could be pivoting. Quite different situation can be observed for $\alpha = 0.6$. In this case party 2 has no influence on outcomes of voting and a cooperation of parties 1 and 3 is needed for approving any proposal.

L.S. Shapley (1953) introduced a solution concept of a cooperative game which is based on an a priori evaluation of power of different players from the point of view of their contribution to all possible coalitions, the so called Shapley vector, or Shapley value.

For a voting game the Shapley vector can be defined as an n–tuple

$$\boldsymbol{h} = (h_1, h_2, \ldots, h_n)$$

such that

$$h_i = \sum_S \frac{(\mid S \mid -1)!(n- \mid S \mid)!}{n!} \tag{3}$$

where $\mid S \mid$ is number of members of coalition S (summation over all winning coalitions such that $i \in S$ and $S - i$ is a losing coalition). It can be shown that

$$\sum_{i \subseteq I} h_i = 1, \qquad h_i \geq 0$$

so $\boldsymbol{h}$ is a probability vector.

Shapley and Shubik (1954) used voting games and suggested the concept of Shapley vector for evaluation of distribution of power in committee systems. In politological literature the value of h_i is called Shapley–Shubik power index for member i of a committee (SS–power index). In fact, h_i is a probability for the i–th party to be essential in transforming a losing coalition into a winning one (for all theoretically possible coalitions).

In our simple example of 3–parties parliament we can define the following voting game (for $\alpha = 0.5$):

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1$$

$$v(\{1,2,3\}) = 1$$

Using (3) we obtain

$$\boldsymbol{h} = (1/3,\ \ 1/3,\ \ 1/3)$$

SS–power index is the same for all the three parties, which corresponds to intuitive reasoning. For $\alpha = 0.6$ we have the following voting game:

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$$

$$v(\{1,2\}) = v(\{2,3\}) = 0$$

$$v(\{1,3\}) = 1$$

$$v(\{1,2,3\}) = 1$$

In this case

$$\boldsymbol{h} = (1/2,\ 0,\ 1/2)$$

which corresponds to an intuitive conclusion that all the power is in the hands of two large parties.

3. Voting games and multi–cameral legislature

In order to be able to explore some particular parliamentary structures, we shall suppose that a "parliament" consists of more "houses" that vote separately and a proposal is approved only if it is approved by each of the "houses". Such a parliamentary structure is called *multi–cameral legislature.*

Let m be the number of "houses" in multi–cameral legislature and a_{ik} be the number of deputies of the i–th party in the k–th house, then

$$a_{0k} = \sum_{i \in S} a_{ik}$$

is the total number of deputies in the k–th house (the number of deputies in different houses need not be the same).

A coalition $S \subseteq I$ is called a winning coalition in a multi–cameral legislature with respect to a voting rule α, if

$$\min_k \left[\sum_{i \in S} a_{ik} - \text{int}(\alpha a_{0k}) \right] > 0$$

(the coalition S should get α–majority in all houses) and a losing coalition in the opposite case (if it fails to get α–majority in at least one house).

Starting from this definition of winning and losing coalitions we can construct in a standard way a voting game for multi–cameral legislature (with characteristic function $v(S) = 1$ for winning coalition and $v(S) = 0$ for losing coalition) and compute corresponding SS–power characterisitics.

To illustrate the concept of multi–cameral legislature and corresponding voting game let us consider the following example of 3–cameral 3–party parliament.

	number of deputies			
parties	house 1	house 2	house 3	$\sum$
1	50	25	10	85
2	45	10	15	70
3	5	15	25	45
$\sum$	100	50	50	200

Possible coalitions, voting outcomes and characteristic function values for $\alpha = 0.5$ and $\alpha = 0.6$ are as follows:

coalitions	number of votes			characteristic function for	
	house 1	house 2	house 3	$\alpha = 0.5$	$a = 0.6$
{1}	50	25	10	0	0
{2}	45	10	15	0	0
{3}	5	15	25	0	0
{1,2}	95	35	25	0	0
{1,3}	55	40	35	1	0
{2,3}	50	25	40	0	0
{1,2,3}	100	50	50	1	1

Let us notice that there are only two winning coalitions for $\alpha = 0.5$, coalition {1,3} and the "grand" coalition {1,2,3}. To compute SS-power index for party 1 we have to find winning coalitions for which party 1 is an essential member. For both of the winning coalitions party 1 is essential: if party 1 withdraws from coalition {1,2,3}, then the remaining coalition {2,3} becomes a losing one (it does not have majority in houses 1 and 2); if party 1 withdraws from coalition {1,3}, then the remaining "coalition" {3} becomes a losing one (it does not have majority in all houses). By the same way we can show that party 3 is essential for both winning coalitions and that party 2 is not essential for the only winning coalition {1,2,3} that it is a member of.

Now we can compute SS–power characteristics for all the three parties. For $\alpha = 0.5$ we have

	$\frac{(\lvert S\rvert-1)!(n-\lvert S\rvert)!}{n!}$		
winning coalitions	party 1	party 2	party 3
{1,3}	1/6	not a member	1/6
{1,2,3}	1/3	not essential	1/3
$\sum$	1/2	0	1/2

hence

$$h = (1/2, 0, 1/2)$$

We received a rather surprising result: with respect to majority rule $\alpha = 0.5$ the power of party 3 (with only 22.5% of deputies) is in our 3–cameral legislature model the same as the power of party 1 (with 42.5% deputies) and due to a specific distribution of votes of different parties among the houses party 2 (with 35% of deputies) has no voting power at all.

For $\alpha = 0.6$ we obtain

$$\boldsymbol{h} = (1/3, 1/3, 1/3)$$

(the same SS–power index for all parties).

It may be of interest to compare the results obtained for our three–cameral parliament with the ones for the standard one–chamber parliament with the same number of deputies and the same distribution of votes among the parties. For $\alpha = 0.5$ we have

$$\boldsymbol{h} = (1/3, 1/3, 1/3)$$

and for $\alpha = 0.6$

$$\boldsymbol{h} = (2/3, 1/6, 1/6)$$

4. The structure of power in Czecho–Slovak parliaments

The parliamentary system in ČSFR consisted of

a) federal parliament with two houses: the house of people and the house of nations, where the house of nations had two separately voting parts — Czech and Slovak;
b) two national parliaments (Czech and Slovak national councils).

The voting rules required more than 50% majority ($\alpha = 0.5$) for standard decisions and procedural questions, and more than 60% majority ($\alpha = 0.6$) for constitutional changes. But simple majority rule was used only in Czech and Slovak parliament. The federal parliament had typical features of multi–cameral legislature: a proposal was approved only if the folowing three conditions wee met:

a) corresponding α–majority in the house of people,
b) α–majority in Czech part of house of nations,
c) α–majority in Slovak part of house of nations.

After the elections of 1990 the following 7 parties entered the federal parliament:

1. *Civic Forum and Public Against Violence — CF/PAV* (Czech and Slovak coalition of liberal centristic movements).
2. *Communist Party — CP* (Czech and Slovak communists).
3. *Movement for Democratic Selfadministration — MDS* (a Moravian national party).
4. *Christian Democratic Union — CDU* (a coalition of Czech christian democrats).
5. *Mutual Understanding Movement — MUM* (a Hungarian national movement).
6. *Christian Democratic Movement — CDM* (Slovak christian democrats).
7. *Slovak National Party — SNP* (radical Slovak nationalist party).

Using multi–cameral approach from section 3 it is possible to construct a voting game for federal parliament with respect to specific voting rules mentioned above (veto in any of three federal parliamentary divisions) and compute corresponding SS–power characteristics (see Table 1).

We can observe that a game–theoretical power distribution differs from distribution of votes: the actual influence of the parties depends on the structure of representation of parties and on voting rules. Relatively more power of the small Slovak parties comparing to small Czech parties follows from "veto–type" voting rules in the house of nations. Increase of the level of α–majority increases the influence of small parties.

Table 1. Structure of political representation and distribution of power in federal parliament of CSFR

Parties	Representation in Federal parliament				Distr. of votes	SS–power index	
	House of people	House of nations Czech	Slovak	$\sum$		$\alpha = 0.5$	$\alpha = 0.6$
CF/PAV	87	50	33	170	0.5667	0.8000	0.6500
CP	23	12	12	47	0.1567	0.0500	0.0667
MDS	9	7	–	16	0.0533	0	0
CDU	9	6	–	15	0.0500	0	0
MUM	5	–	7	12	0.0400	0.0500	0.0667
SNP	6	–	9	15	0.0500	0.0500	0.0667
CDM	11	–	14	25	0.0833	0.0500	0.1500
	150	75	75	300	1.0000	1.0000	1.0000

Table 2 gives *SS*–power characteristics for parties represented in the Czech parliament (Czech National Council). Thus, the *Civic Forum movement* has complete control over Czech parliament, since it has sufficient number of deputies (63.5%) to approve any proposal both in case of $\alpha = 0.5$ and of $\alpha = 0.6$.

A more interesting situation occurs in the Slovak parliament (Slovak National Council) where 7 parties are represented (besides the parties mentioned before there are two other small parties represented in SNC — the *Green Party* and the *Democratic Party*). *SS*–power characteristics for parties represented in the Slovak parliament are given in Table 3.

Table 2. Structure of political representation and distribution of power in the Czech National Council

Parties	Number of deputies	Distribution of votes	*SS*–power index	
			$\alpha = 0.5$	$\alpha = 0.6$
CF	127	0.635	1.000	1.000
CP	32	0.160	0	0
MDS	22	0.110	0	0
CDU	19	0.095	0	0
	200	1.000	1.000	1.000

Table 3. Structure of political representation and distribution of power in the Slovak National Council

Parties	Number of deputies	Distribution of votes	*SS*–power index	
			$\alpha = 0.5$	$\alpha = 0.6$
PAV	48	0.320	0.4119	0.3833
CDM	31	0.206	0.1952	0.1833
CP	22	0.147	0.1286	0.1333
SNP	22	0.147	0.1286	0.1333
MUM	14	0.093	0.0452	0.1000
DP	7	0.047	0.0452	0.0500
Greens	6	0.040	0.0452	0.0167
	150	1.000	1.000	1.000

We can see that no party in SNC had sufficient support to control voting without broad coalitional cooperation.

Data (number of deputies for different parliamentary parties) were taken from final results of the June 1990 parliamentary election. However, the real situation has dramatically changed during 1991. Differentiation in former voting parties and coalitions led to fast diversification of political formations (created by splitting the original parties) in Federal Parliament at the end of 1991, so that the list would be (we give only very "unofficial" translation of the parties or factions names):

Movement for Democratic Selfadministration I (MDS/I);
Movement for Democratic Selfadministration II (MDS/II) not organized (NO);
Social Democratic Orientation, part of the former Civic Forum (SDO);
Christian Democratic Union (CDU);
Czech and Moravian Communist Party (CMCP);
Party of Democratic Left, former Slovak Communist Party (PDL);
Civic Movement, part of the former Civic Forum (CM);
Civic Democratic Party, part of the former Civic Forum (CDP);
Civic Forum — independent, part of the former Civic Forum (CFI);
Christian Democratic Party and Liberal Democratic Party, Czech (CDP/LDP);
Mutual Understanding Movement and Hungarian Christian Democrats (MUM/HCD);
Civic Democratic Aliance, part of the former Civic Forum (CDA);
Civic Democratic Union, part of the former Public Against Violence (CDU/PAV);
Christian Democratic Movement, Slovak (CDM);
Slovak National Party (SNP);
Movement for Democratic Slovakia, part of the former Public Against Violence (MFDS).

In Table 4 we give the changed structure of the Federal Parliament of Czech and Slovak Republic (number of deputies, relative distribution of votes and values of power indices for parliamentary political formations).

Table 4. Changes in distribution of power in CSFR Federal Parliament (end of 1991)

parties (factions)	votes in Federal Parliament								power index	
	HP		HNC		HNS		Σ		$\alpha = 0.5$	$\alpha = 0.6$
	votes	%	votes	%	votes	%	votes	%		
MDS/I	6	4.00	4	5.33	0	0.00	10	3.33	2.38	3.22
MDS/II	3	2.00	3	4.00	0	0.00	6	2.00	1.68	2.29
NO	4	2.67	1	1.33	3	4.00	8	2.67	3.79	2.37
SDO	5	3.33	4	5.33	0	0.00	9	3.00	2.33	3.12
CDU	7	4.67	4	5.33	0	0.00	11	3.67	2.43	3.33
CMCP/PDL	23	15.33	12	16.00	12	16.00	47	15.67	17.39	12.47
CM	24	16.00	17	22.67	0	0.00	41	13.67	12.22	12.41
CDP	26	17.33	18	24.00	0	0.00	44	14.67	13.54	13.46
CFI	4	2.67	3	4.00	0	0.00	7	2.33	1.73	2.38
CDP/LDP	3	2.00	3	4.00	0	0.00	6	2.00	1.68	2.29
MUM/HCDP	5	3.33	0	0.00	5	6.67	10	3.33	3.33	2.32
CDA	5	3.33	6	8.00	0	0.00	11	3.67	3.31	5.13
CDU–PAV	11	7.33	0	0.00	23	30.67	34	11.33	15.97	16.12
CDM	11	7.33	0	0.00	14	18.67	25	8.33	8.29	7.80
SNP	5	3.33	0	0.00	9	12.00	14	4.67	4.90	5.35
MFDS	8	5.33	0	0.00	9	12.00	17	5.67	5.02	5.70
Σ	150	100.0	75	100.0	75	100.0	300	100.0	100.0	100.0

The same tendency to "dissipation" of Czechoslovak political scene can be observed both in the Czech National Council (formerly 4 parties, now 11 parties and factions) and the Slovak National Council (formerly 7 parties, now 11 parties and factions).

5. Concluding remarks

A game–theoretical analysis of an a priori power distribution gives a deeper insight into a set of political possibilities that could occur in the representative bodies (parliaments, committees etc.). This can be useful for an evaluation of possible governmental coalitions after election. New voting rules and procedural rules designed for committees should be tested from the point of view of real power distribution and its correspondence to the distribution of votes on the one side, and from the point of view of protection of minorities on the other side.

A sensitivity analysis of a power distribution with respect to small changes of the distribution of votes could give some characteristics of political stability of representative bodies and government coalitions.

A week point of the Shapley–Shubik power index as a measure of real power distribution follows from the assumption that all the theoretically possible coalitions are equally probable. In fact, there can exist some highly improbable coalitions (e.g. comunists and christian democrats, Hungarian national movement and Slovak national party) and a list of possible coalitions should take into account such restrictions.

An alternative measure of power distribution was suggested by J.F.Banzhaf in 1965. The Banzhaf power index (B–power index) for a party i is defined as the ratio

$$\frac{c_i}{\sum_{i\in I} c_i}$$

where c_i is a number of winning coalitions for which the i–th party is essential. In the general case the B–power index could differ from the SS–power index. B–power index gives a probability with which the i–th party has the "blocking power" (i.e. it is able to destroy the winning coalitions). If it is possible to define a set of feasible coalitions (excluding highly improbable coalitions), then we can compute B–power index taking into account only the "feasible" winning coalitions.

Democratic voting procedures are used not only in parliaments and other political bodies, but also in councils of shareholders in joint–stock companies, where (usually) the "number of votes" of a shareholder is given by his capital share. A voting power is then measured by the capital power. We can see that it need not be necessarily so; the real voting power of a shareholder in corporate decision making does not reduce to his capital strength.

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